INFLUENCE OF MAGNETIC VISCOSITY ON THE DEVELOPMENT OF INSTABILITY OF THE BOUNDARY OF A PLASMA BLOB IN A UNIFORM MAGNETIC FIELD

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Studying the dynamics of plasma blobs moving in a magnetic field is of interest for the solution of a number of problems in astrophysics [1] and geophysics [2, 3], as well as for interpreting experiments with a laboratory plasma [4-6].

Many authors have considered the problem of the dispersal of a plasma cloud in a magnetic field (see, e.g., [7-13]). To summarize their results, it can be concluded that fairly complete concepts have now been developed about the nature of the motion of plasma blobs in a magnetic field. A combined kinetic and hydrodynamic model, leading to cumbersome numerical calculations, has been used in a number of papers [11, 12]. The important question of the instabilities that characterize the dynamics of a plasma cloud in a magnetic field remains insufficiently studied, since its analysis is associated with further complication of the mathematical model. The main results on the development of magnetohydrodynamic (MHD) instabilities have been obtained, as a rule, from the analysis of dispersion relations (see [14]).

The present paper has the aim of formulating a model of the dynamics of a plasma cloud in a magnetic field that is fairly simple for mathematical analysis, on the one hand, and complete enough that features of the development of MHD instabilities of the cloud surface can be described within its framework, on the other. The model of a plasma cloud, filled with a weakly inhomogeneous, collisionless, magnetized plasma, is used here. To describe the plasma we use the Chu-Goldberger-Low (CGL) model [14] with allowance for corrections associated with the finite Larmor radius of the ions, leading to the appearance of viscosity in the equations [15]. The boundary of the plasma cloud is described mathematically as a discontinuity in the fields and plasma parameters. The problem of the boundary's motion can be formulated so that only parameters characterizing the boundary itself appear in it. This enables us to use the method of contour dynamics to describe the instability of the boundary between plasma and vacuum when the surface is curved. The corresponding instability is called a flute or transposition instability; the conditions under which it develops have been analyzed in [16]. Allowance for gyroviscosity has made it possible to explain the experimentally observed spatial spectrum of surface instabilities of a plasma blob, and has established that the flutings travel along the plasma surface across the magnetic field.

1. Equations. The plasma is described in the CGL approximation by the system of equations [14]

$$\frac{d\rho}{dt} + \rho \operatorname{div} \mathbf{V} = 0, \quad \frac{d}{dt}\frac{p_\perp}{\rho B} = 0, \quad \frac{d}{dt}\frac{p_\parallel B^2}{\rho^3} = 0,$$
$$\rho \frac{d\mathbf{V}}{dt} = -\operatorname{div} \mathbf{P} + \frac{1}{c}(\mathbf{j} \times \mathbf{B}), \; \mathbf{E} + \frac{1}{c}(\mathbf{V} \times \mathbf{B}) = 0, \tag{1.1}$$

where ρ is the plasma density; $\mathbf{P}$ is the pressure tensor, and components of which depend on $p_\perp$ and $p_\parallel$, which characterize the plasma pressure across and along the magnetic field.

The electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are determined from the solution of the Maxwell equations for a quasi-neutral plasma:

$$\operatorname{curl} \mathbf{B} = \frac{4\pi}{c}\mathbf{j}, \; \operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \; \operatorname{div} \mathbf{B} = 0, \; \operatorname{div} \mathbf{E} = 0. \tag{1.2}$$

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To investigate the instability of the boundary of a magnetized plasma, one must integrate Eqs. (1.1) and (1.2) in the cloud with a free surface. Since such a problem is extremely complicated, it is expedient to adopt a number of simplifying assumptions.

First, to study the evolution of flute perturbations, we confine ourselves to considering the central cross section of the cloud, transverse to the external magnetic field (Fig. 1). The three-dimensional problem then reduces to a planar one. Assuming the cloud to be symmetric relative to the central cross section, we note that the external field $\mathbf{B}_0$ and the field $\mathbf{B}$ within the plasma are orthogonal to the plane of the cross section. For the current and the velocity of particles lying in the plane under consideration we then find, from the last equations of system (1.1),

$$\mathbf{j} = \frac{c}{B^2}\mathbf{B}\times\left(\rho\frac{d\mathbf{V}}{dt} + \operatorname{div}\mathbf{P}\right), \quad \mathbf{V} = \frac{c}{B^2}(\mathbf{E}\times\mathbf{B}). \tag{1.3}$$

We resolve the magnetic field in the plasma into a constant and uniform part $\mathbf{B}_1$ and a variable correction $\mathbf{B}_1'$. Substituting $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_1'$ into the first equation of (1.2) and using the expression from (1.3) for the current, we easily show that the correction B_1' is small compared with B_1 if the particle drift velocity V is considerably lower than the Alfvén velocity $V_a = B/\sqrt{4\pi\rho}$ and the pressure of the plasma particles is considerably lower than the magnetic pressure. Since these conditions are typical of a late stage of dispersal, we assume the magnetic field inside the plasma to be constant. The jump in the field at the surface of the cloud (the transition from B_0 to B_1) is determined by the surface current flowing along the loop L bounding the central cross section, while the volume currents distort the magnetic field $\mathbf{B}_1$ little.

The assumption that the field is constant means that the equalities curl $\mathbf{B}$ = 0 and curl $\mathbf{E}$ = 0 are satisfied inside the loop, which entails the condition of incompressibility (div $\mathbf{V}$ = 0) for the plasma. Then assuming that ρ, $p_\perp$, and $p_\parallel$ are initially constant in the cloud's central cross section, from Eqs. (1.1) we find that they also remain constant over all subsequent time intervals. These facts stimulate further simplification of the problem's formulation: Equations are derived for the evolution of the loop L as a mathematical surface of discontinuity in fields and plasma parameters. The problem of the boundary's position can be formulated so that only parameters characterizing the boundary itself appear in it. This enables one to reduce the analysis of the development of instability of the surface of a plasma cloud to the solution of a one-dimensional, nonsteady problem.

The motion of the boundary of the central cross section is defined if at each point on it one knows the normal velocity D, which coincides with the component of V_n of the drift velocity normal to the boundary:

$$D = -cE_\tau B_1. \tag{1.4}$$

Hence, it follows that the problem consists in finding the components E_τ and E_n of the electric field that are induced by the boundary's motion and depend on the buildup of surface charge on it (the subscripts n and τ mark projections onto the outward normal and tangent to the boundary L of the central cross section).

Let the jump $[B] = B_0 - B_1$ in magnetic field strength be given at L and let there be a surface charge σ. The electric field [17] will then undergo a discontinuity at the contour,

$$[E_\tau] = -Dc^{-1}[B], [E_n] = 4\pi\sigma, [E_\tau] = E_\tau^+ - E_\tau^-, [E_n] = E_n^+ - E_n^-,$$

where the + (−) sign marks the region external (internal) to the loop L. We represent the field as a superposition $\mathbf{E} = \mathbf{\Psi} + \mathbf{\Phi}$ of the vortical and polarization fields, for which we have

$$\operatorname{curl}\ \mathbf{\Psi} = 0,\ \operatorname{div}\mathbf{\Psi} = 0,\ \operatorname{curl}\ \mathbf{\Phi} = 0,\ \operatorname{div}\mathbf{\Phi} = 0 \tag{1.5}$$

outside the boundary and

$$[\Psi_\tau] = -Dc^{-1}[B],\ [\Psi_n] = 0,\ [\Phi_\tau] = 0,\ [\Phi_n] = 4\pi\sigma \tag{1.6}$$

at the boundary.

We introduce the stream function ψ and the potential φ: $\mathbf{\Psi} = -\mathbf{b}\times\operatorname{grad}\psi$, $\mathbf{\Phi} = -\operatorname{grad}\varphi$, $\mathbf{b} = \mathbf{B}/B$, from which, using (1.5) and by virtue of the flow's two-dimensionality, we have

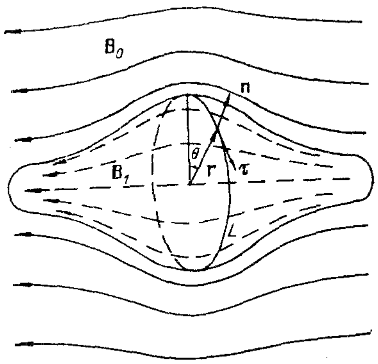

Fig. 1

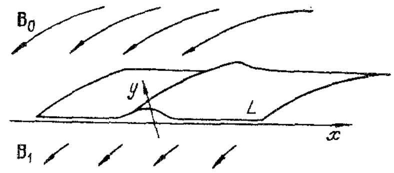

Fig. 2

$$\Delta\psi = 0,\ \Delta\varphi = 0. \tag{1.7}$$

Since we have

$$\Psi_\tau = \frac{\partial\psi}{\partial\mathbf{n}},\ \Psi_n = -\frac{\partial\psi}{\partial\boldsymbol{\tau}},\ \Phi_\tau = -\frac{\partial\varphi}{\partial\boldsymbol{\tau}},\ \Phi_n = -\frac{\partial\varphi}{\partial\mathbf{n}}, \tag{1.8}$$

from (1.6) we obtain the conditions for the functions ψ and φ at the boundary L:

$$\left[\frac{\partial\psi}{\partial\mathbf{n}}\right] = -\frac{D}{c}[B],\ \left[\frac{\partial\psi}{\partial\boldsymbol{\tau}}\right] = 0,\ \left[\frac{\partial\varphi}{\partial\mathbf{n}}\right] = -4\pi\sigma,\ \left[\frac{\partial\varphi}{\partial\boldsymbol{\tau}}\right] = 0. \tag{1.9}$$

We have thus formulated the problems (1.7) and (1.9) on conjugation at L of harmonic functions with a continuous tangential normal derivative and one that undergoes a specific discontinuity. We write the conditions for these problems to be solvable for sufficiently smooth L in the form

$$\oint_L D\,ds = 0,\quad \oint_L \sigma\,ds = 0. \tag{1.10}$$

We seek solutions to the problems (1.7) and (1.9) in the form of the logarithmic potential of a simple layer with density μ (s' is the natural parameter of the curve and $\mathbf{r}'$ is the vector corresponding to the point s'):

$$Q(\mathbf{r}) = \oint_L \mu(\mathbf{r}') \ln\frac{1}{\delta r}\,ds',\ \delta r = |\mathbf{r}' - \mathbf{r}|.$$

We write the well-known [18] expressions for the boundary values of the normal derivative of the potential Q:

$$\frac{\partial Q^+(\mathbf{r})}{\partial\mathbf{n}} = -\pi\mu(\mathbf{r}) + \oint_L \mu(\mathbf{r}')\frac{\cos\alpha}{\delta r}\,ds',$$
$$\frac{\partial Q^-(\mathbf{r})}{\partial\mathbf{n}} = \pi\mu(\mathbf{r}) + \oint_L \mu(\mathbf{r}')\frac{\cos\alpha}{\delta r}\,ds' \tag{1.11}$$

(α is the angle between the vectors $\mathbf{r}' - \mathbf{r}$ and $\mathbf{n}$). We then have

$$[\partial Q/\partial\mathbf{n}] = -2\pi\mu. \tag{1.12}$$

The derivative $\partial Q/\partial\tau$ along the direction tangent to the boundary is continuous, according to [18] (β is the angle between $\mathbf{r}' - \mathbf{r}$ and τ):

$$\frac{\partial Q(\mathbf{r})}{\partial\tau} = \oint_L \mu(\mathbf{r}')\frac{\cos\beta}{\delta r}\,ds'. \tag{1.13}$$

The conditions (1.12) and (1.13) are consistent with (1.9) for $\mu_\psi = D[B]/(2\pi c)$ and $\mu_\varphi = 2\sigma$.

Using these relations for the density of "sources" producing the fields $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$, and using Eqs. (1.8), (1.11), and (1.13), we find the components of the field $\mathbf{E}$ at the boundary as it is approached from the plasma side:

$$E_\tau^- = \frac{D[B]}{2c} + \frac{[B]}{2\pi c}\oint_L D\frac{\cos\alpha}{\delta r}\,ds' - 2\oint_L \sigma\frac{\cos\beta}{\delta r}\,ds',$$

$$E_n^- = -2\pi\sigma - \frac{[B]}{2\pi c}\oint_L D\frac{\cos\beta}{\delta r}\,ds' - 2\oint_L \sigma\frac{\cos\alpha}{\delta r}\,ds'. \tag{1.14}$$

Equation (1.4) supplements Eqs. (1.14) by relating E_τ with D. To close the system of equations, we must express the charge σ in terms of the components of the field **E**. We use the equation of charge conservation

$$\partial\sigma/\partial t = j_n, \tag{1.15}$$

connecting the surface charge density with the normal component j_n of the current to the boundary. We find the expression for j_n from Eq. (1.3), written at the boundary of the cloud's central cross section in projection onto the normal to the loop L. Substituting the result obtained with allowance for the uniformity of the distribution of plasma parameters within the cloud into Eq. (1.15), we find

$$\frac{\partial\sigma}{\partial t} = \frac{c^2}{B_1^2}\rho\frac{\partial E_n^-}{\partial t} + \frac{c}{B_1}(\operatorname{div}\mathbf{P})_\tau,$$

which supplements Eqs. (1.4) and (1.14) to close the system.

We eliminate E_n and E_τ from this system, use the fact that $V_a/c \ll 1$, and replace σ by the new variable $\Sigma = 4\pi c\sigma/(B_0 + B_1)$. The result is a system of equations for the two unknowns D and Σ:

$$D(t,\mathbf{r}) = \frac{-[B]}{\pi(B_0+B_1)}\oint_L D\frac{\cos\alpha}{\delta r}\,ds' + \oint_L \Sigma\frac{\cos\beta}{\delta r}\,ds',$$

$$\frac{\partial\Sigma(t,\mathbf{r})}{\partial t} = \frac{2B_1(\operatorname{div}\mathbf{P})_\tau}{\rho(B_0+B_1)} - \frac{1}{\pi}\frac{\partial}{\partial t}\left\{\frac{[B]}{B_0+B_1}\oint_L D\frac{\cos\beta}{\delta r}\,ds' + \oint_L \Sigma\frac{\cos\alpha}{\delta r}\,ds'\right\}. \tag{1.16}$$

If we know $(\operatorname{div}\mathbf{P})_\tau$ at the boundary, we can determine D(t, **r**) and hence describe the evolution of L. In general, we have (see [14])

$$(\operatorname{div}P)_\tau = \{\operatorname{grad}p_\perp + (p_\parallel - p_\perp)(\mathbf{b}\operatorname{grad})\mathbf{b}\}_\tau. \tag{1.17}$$

The relationship between grad $p_\perp$ and the curvature vector $(\mathbf{b}\operatorname{grad})\mathbf{b}$ at the plasma boundary can be found from the condition that the normal component of the magnetization current vanish at it [19]. Since $p_\perp/B^2 \ll 1$, for the central cross section of the cloud we have

$$(\mathbf{j}_m)_\perp = -c\operatorname{curl}(p_\perp\mathbf{b}/B) = -c\mathbf{b}/B \times \{2p_\perp(\mathbf{b}\operatorname{grad})\mathbf{b} - \operatorname{grad}p_\perp\}.$$

Determining $(j_m)_n$ from this, equating it to zero, and using (1.17), we finally determine that at the boundary we have

$$(\operatorname{div}\mathbf{P})_\tau = \{(p_\perp + p_\parallel)(\mathbf{b}\operatorname{grad})\mathbf{b}\}_\tau. \tag{1.18}$$

2. Viscosity. The existence of solutions of the type $\exp(-i\omega t + i\mathbf{kr})$, with the instability increment $\gamma = \operatorname{Im}(\omega)$ increasing without limit as the wave number k increases, has been found in the analysis of plasma instability based on the CGL approximation [14]. The departures of the properties of the plasma from those of its MHD model become really significant at large k. The main cause of such departures is obvious: the finite Larmor radius R_i of the ions.

As shown in [15], allowance for the finiteness of R_i in the CGL model leads to a representation of the pressure tensor in the form $\mathbf{P} = \mathbf{p} + \mathbf{q}$, where the tensor **p** corresponds to the anisotropic pressure in the plasma while the tensor **q** characterizes viscous corrections of order $1/\omega_i$ (ω_i is the ion cyclotron frequency). A general expression for **q** has been obtained in [20]. Since the lines of the external magnetic field, displaced by the plasma, have a constant radius of curvature R_b, and neglecting terms of order $1/(\omega_i R_b)$ and $1/R_b^2$ we can use (1.18) to obtain from [20] an expression for $(\operatorname{div}\mathbf{P})_\tau = (\operatorname{div}\mathbf{q})_\tau + (\operatorname{div}\mathbf{p})_\tau$ at the boundary of the cloud's central cross section:

$$(\operatorname{div}\mathbf{P})_\tau = -\frac{p_\perp}{2\omega_i}\frac{\partial^2 D}{\partial s^2} - \frac{p_\perp + p_\parallel}{R_b}\cos\eta. \tag{2.1}$$

Here η is the angle between the radius vector **r**, drawn from the cloud's axis toward some point of the loop L, and the tangent vector **τ** at the same point. Substituting (2.1) into (1.16) completes the derivation of the closed system.

3. Model Problem. Let us consider a flute instability of a surface, curved along the field, that is confined to the part of space filled with plasma (Fig. 2). Let the magnetic flux lines have a constant radius of curvature R_b, let the vectors $\mathbf{B}_0$ and $\mathbf{B}_1$ be collinear, and let a discontinuity $[B] = B_0 - B_1$ be specified at the surface.

At the intersection of space with the plane orthogonal to the magnetic flux lines, the infinitely extended boundary L separating plasma and vacuum starts to deform under the influence of the drift of charged particles. In the Cartesian coordinate system with the x axis along the unperturbed boundary, the curve L will be described by the equation $y = Y(t, x)$. The velocity of the boundary's motion along the normal is defined by

$$D(t,x) = \frac{\partial Y(t,x)}{\partial t}\left\{1 + \left(\frac{\partial Y}{\partial x}\right)^2\right\}^{-1/2}. \tag{3.1}$$

Let us consider small perturbations $y = Y(t, x)$ of the boundary. Assuming Y, $\partial Y/\partial x$, $\partial Y/\partial t$, Σ, $\partial \Sigma/\partial t$, and $Y(t, x') - Y(t, x)$ to be of first-order smallness, and neglecting their squares and products, from (1.16), (2.1), and (3.1) we obtain the linearized system of equations

$$\frac{\partial \Sigma}{\partial t} = \frac{-2B_1}{B_0 + B_1}\left(G\frac{\partial Y}{\partial x} + \nu\frac{\partial^3 Y}{\partial t\,\partial x^2}\right) - \frac{B_0 - B_1}{\pi(B_0 + B_1)}\frac{\partial}{\partial t}\int_{-\infty}^{+\infty}\frac{\partial Y}{\partial t}\frac{dx'}{x' - x},$$

$$\frac{\partial Y}{\partial t} = \frac{1}{\pi}\int_{-\infty}^{+\infty}\Sigma(t,x')\frac{dx'}{x' - x},\quad G = \frac{p_\parallel + p_\perp}{\rho R_b},\quad \nu = \frac{p_\perp}{2\rho\omega_i},$$

where the parameters G and ν have the dimensions of acceleration and viscosity.

From the system's second equation, it follows that the function $\partial Y/\partial t$ is conjugate to Σ with respect to a Hilbert transform with a Cauchy kernel [21]. Using the inverse transform, we can express the integral on the right side of the first equation in terms of the function Σ. After simple manipulations, we have

$$\frac{\partial Y}{\partial t} = \frac{1}{\pi}\int_{-\infty}^{+\infty}\Sigma(t,x')\frac{dx'}{x' - x},\quad \frac{\partial \Sigma}{\partial t} = -G\frac{\partial Y}{\partial x} - \nu\frac{\partial^3 Y}{\partial t\partial x^2}. \tag{3.2}$$

The stability of the system (3.2) against perturbations of the type $\exp(-i\omega t + ikx)$ is determined by the dispersion relation

$$\omega^2 + \nu k^2\omega + Gk = 0. \tag{3.3}$$

It is easy to see that for $\nu = 0$ the system is unstable, with the instability increment $\gamma(k)$ increasing without limit as the wave number grows. For $\nu \neq 0$, perturbations grow only for $0 < k < k_0$, where $k_0 = (4G/\nu^2)^{1/3}$. The increment reaches its maximum $\gamma_* = \sqrt{3}/2(G^2/\nu)^{1/3}$ at $k = (G/\nu^2)^{1/3}$. Allowance for gyroviscosity thus leads to stabilization of short-wavelength modes. Perturbations grow in a wavelength range considerably exceeding the Larmor radius,

$$\lambda > \lambda_0 = \pi/k_0 = \pi\{\nu^2/(4G)\}^{1/3} \simeq R_i(R_b/R_i)^{1/3} \gg R_i,$$

which indicates the applicability of the above approach to instability analysis. We should note that the solution to Eq. (3.3) also has a real part that depends on k. This leads to the effect of wave propagation of the initial perturbations, which is also determined by the presence of gyroviscosity. Long-wavelength perturbations travel with phase velocity $-\nu k/2$.

Let us consider flute perturbations of the "wave packet" type:

$$Y(t = 0,\ x) = Y_0/\{1 + (x/x_0)^2\},\ \Sigma(t = 0,\ x) = 0. \tag{3.4}$$

For $\nu = 0$ the problem (3.2), (3.4) can be solved using a Fourier integral transform

$$Y(t,x) = Y_0 x_0\int_0^\infty \exp(-kx_0)\,\mathrm{ch}\left(t\sqrt{Gk}\right)\cos(kx)\,dk. \tag{3.5}$$

For large x we can obtain the asymptotic form of the solution:

$$Y(t, x) \simeq \frac{Y_0 x_0}{x^2}\left(x_0 - \frac{Gt^2}{2}\right).$$

At x = 0 the integral (3.5) can be calculated:

$$Y(t, x = 0) = Y_0\left\{\frac{t\sqrt{G}}{2}\sqrt{\frac{\pi}{x_0}}\exp\left(\frac{Gt^2}{4x_0}\right)\operatorname{erf}\left(\frac{t\sqrt{G}}{2\sqrt{x_0}}\right) + 1\right\}. \tag{3.6}$$

One can see from (3.6) that the flute perturbation (3.4) develops considerably faster than a sinusoidal perturbation and the amplitude buildup accelerates with decreasing pulse width x_0.

We introduce the scales of length Y_0, time $\sqrt{Y_0/G}$, and charge $\sqrt{Y_0 G}$. The problem (3.2), (3.4) in dimensionless variables takes the form

$$\begin{aligned} &\frac{\partial Y}{\partial t} = \frac{1}{\pi}\int\limits_{-\infty}^{+\infty}\Sigma(t, x')\frac{dx'}{x' - x}, \quad Y(t = 0, x) = \frac{1}{(1 + xd)^2}, \\ &\frac{\partial \Sigma}{\partial t} = -\frac{\partial Y}{\partial x} - \frac{1}{\mathrm{Re}}\frac{\partial^3 Y}{\partial t \partial x^2}, \ \Sigma(t = 0, x) = 0, \end{aligned} \tag{3.7}$$

where $\mathrm{Re} = Y_0\sqrt{Y_0 G}/\nu$ is the analog of the Reynolds number; $d = Y_0/x_0$ is the "quality" of the initial pulse.

Equation (3.5) can be transformed similarly; the results of a calculation based on it for d = 1 are given in Fig. 3. Curves 1 and 2 correspond to the boundary's position at t = 0 and 2. A pattern of development of the instability that is symmetric with respect to the x axis is observed. It is seen that a negative phase develops at a certain time; the calculation agrees well with the asymptotics.

For $\nu \neq 0$, the problem (3.7) is investigated by the finite-difference method [22]. The calculated results are also given in Fig. 3, in which it is shown, for Re = 2 and d = 1, how the plasma boundary deforms by t = 2 (curve 3). Allowance for gyroviscosity leads to interesting effects. It is seen that the perturbation amplitude does not grow as rapidly as in its absence (see for curve 2), with the axis of greatest growth of the perturbation shifting leftward from its initial position. A negative phase develops along with the positive one, as before, but the wave structure loses symmetry. The process of charge separation at the plasma boundary is illustrated in Fig. 3 ($\nu \neq 0$, curve 4), from the initial time, when $\Sigma = 0$, to the time t = 2. It is seen that the positive and negative charges are concentrated on different slopes of the hills and valleys formed at the boundary. The local electric field generated in the process stimulates the instability's development.

We note one other fact. Calculations of (3.7) with Re fixed and d varied showed that a certain "quality" of the initial pulse (3.4) exists for which the amplitude of the perturbation grows at the highest rate. This accords with the fact that a wavelength of a sinusoidal perturbation λ_* exists that corresponds to the largest growth increment.

4. Evolution of the Boundary of the Central Cross Section of a Spindle-Shaped Cloud. Let the loop L be given by the equation $r = R(t, \theta)$ in polar coordinates r, θ (see Fig. 1). Since

$$D = \frac{\partial R}{\partial t}\left\{1 + \left(\frac{1}{R}\frac{\partial R}{\partial \theta}\right)^2\right\}^{-1/2}, \quad \cos\eta = \frac{\partial R}{\partial \theta}\left\{R^2 + \left(\frac{\partial R}{\partial \theta}\right)^2\right\}^{-1/2},$$

from (1.16) and (2.1) we obtain a linearized system of equations for the small quantities R_1 and Σ_1 ($R = R_0 + R_1$, $\Sigma = \Sigma_1$):

$$\begin{aligned} &\frac{\partial R_1}{\partial t} = \frac{1}{2\pi}\frac{B_0 - B_1}{B_0 + B_1}\int\limits_0^{2\pi}\frac{\partial R_1}{\partial t}d\theta + \frac{1}{2\pi}\int\limits_0^{2\pi}\Sigma_1\cot\frac{\theta' - \theta}{2}d\theta', \\ &\frac{\partial \Sigma_1}{\partial t} = \frac{1}{2\pi}\frac{\partial}{\partial t}\left\{\int\limits_0^{2\pi}\Sigma_1 d\theta - \frac{B_0 - B_1}{B_0 + B_1}\int\limits_0^{2\pi}\frac{\partial R_1}{\partial t}\cot\frac{\theta' - \theta}{2}d\theta'\right\} - \\ &-\frac{2B_1}{B_0 + B_1}\left\{\frac{G}{R_0}\frac{\partial R_1}{\partial \theta} + \frac{\nu}{R_0^2}\frac{\partial^3 R_1}{\partial t \partial \theta^2}\right\}, \quad R_1(t, 0) = R_1(t, 2\pi), \ \Sigma_1(t, 0) = \Sigma_1(t, 2\pi). \end{aligned} \tag{4.1}$$

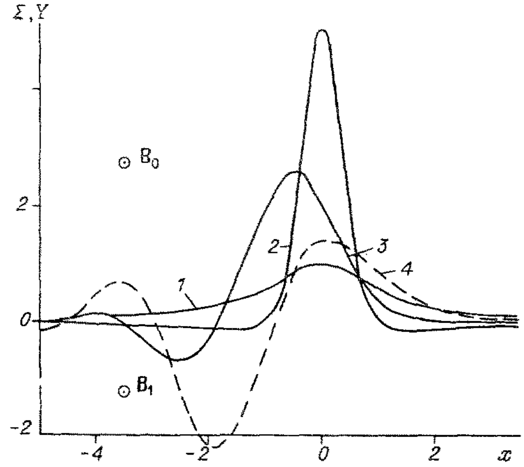

Fig. 3

Using the formula for inverting a Hilbert transform [21] and the solvability conditions (1.10), from the first equation of (4.1) we find

$$\Sigma_1 = -\frac{1}{2\pi}\int_0^{2\pi} \frac{\partial R_1}{\partial t} \cot\frac{\theta' - \theta}{2}\, d\theta'. \tag{4.2}$$

Introducing the scales of length R_0 and time $(R_0/G)^{1/2}$ and using (4.2), we can rewrite the system (4.1) in the dimensionless form

$$\frac{\partial R_1}{\partial t} = \frac{1}{2\pi}\int_0^{2\pi} \Sigma_1 \cot\frac{\theta' - \theta}{2}\, d\theta', \quad \frac{\partial \Sigma_1}{\partial t} = -\frac{\partial R_1}{\partial \theta} - \frac{1}{\mathrm{Re}}\frac{\partial^3 R_1}{\partial t \partial \theta^2}. \tag{4.3}$$

It is easy to write the dispersion relation for the system (4.3)

$$\omega^2 + \omega k^2/\mathrm{Re} + k = 0, \quad \omega = w + i\gamma, \quad k \in Z,$$
$$w = -k^2/(2\,\mathrm{Re}), \quad \gamma = (k - w^2)^{1/2}, \quad k_* = \mathrm{Re}^{2/3} \tag{4.4}$$

and its exact solution for a sinusoidal initial perturbation:

$$R_1 = A \exp(\gamma t) \cos(wt - k\theta), \ \sin\varepsilon = -w/|\omega|,$$
$$\Sigma_1 = A|\omega| \exp(\gamma t) \sin(wt - k\theta + \varepsilon), \ \cos\varepsilon = -\gamma/|\omega|. \tag{4.5}$$

The development of a periodic system of planar jets at the boundary of the plasma blob is observed in accordance with (4.5). The pattern of charge distribution is noticeably phase-shifted. The entire configuration rotates counterclockwise (if viewed from the end of the **B** vector) an angular velocity $w/k = -0.5k/\mathrm{Re}$. A numerical calculation of (4.3) for $A = 0.1$ and $\mathrm{Re} = 10$ ($k_* = 4$) shows good agreement with (4.5) [22].

Let us consider the evolution of a single fluting, defined initially by the equations

$$R_1 = A/\{1 + (\theta/\theta_0)^2\}, \ \Sigma_1 = 0, \tag{4.6}$$

where the parameter θ_0 characterizes the "quality" of the pulse. In Fig. 4 we show the results of a numerical solution of the system (4.3) for $\mathrm{Re} = 10$, $A = 0.1$, and $\theta_0 = 3\pi/40$. The circle 1 corresponds to the unperturbed boundary, curve 2 to the initial perturbation, and 3 to the deformation of the surface [the function $R(\theta)$] at $t = 2$. The perturbation is seen to evolve in accordance with previously established laws. The positive phase develops rapidly with the formation of a planar jet; a negative phase appears that is absent from the initial profile. Curve 4 was constructed from the function $\Sigma(\theta)$ at $t = 2$. The points on the curve corresponding to positive (negative) charge were offset from the circle 1 on the outer (inner) parts of straight lines $\theta = \mathrm{const}$. Charge separation at the surface occurs so that opposite charges are concentrated on different slopes of hills and valleys. The resulting local electric field (shown by arrows) contributes to further development of the instability. It should be noted that the fluting, deforming with time, travels counterclockwise as seen from the end of the **B** vector.

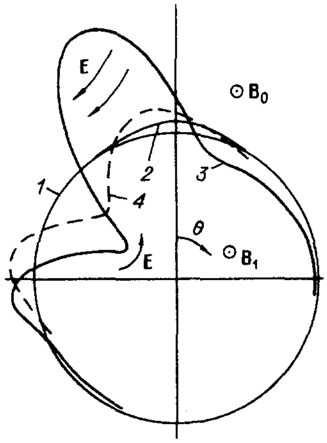

Fig. 4

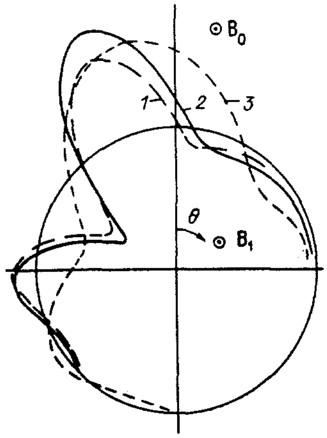

Fig. 5

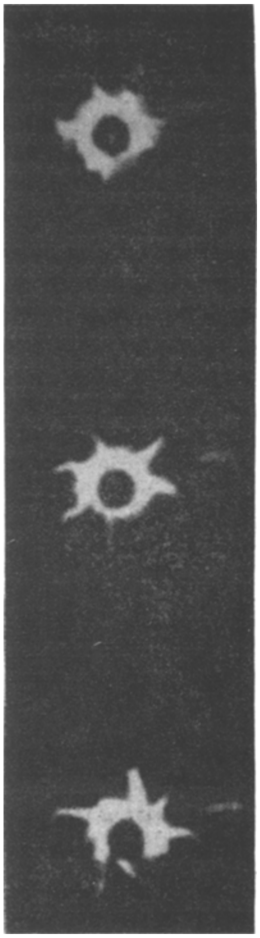

Fig. 6

In Fig. 5 we give the results of a calculation of several versions of the development of the perturbation (4.6), differing in the value of θ_0. The positions of the boundary are shown at t = 2. The lines 1-3 correspond to θ_0 = h, 2h, 6h (h = $\pi/32$). The calculations demonstrate the selective nature of development of an instability that appears with allowance for gyroviscosity. It is seen that the perturbation corresponding to θ_0 = 2h develops most intensively. The configuration corresponding to the fastest growing perturbation mode usually occurs in practice.

5. Comparison with Experiment. Let us consider experimental data [4] on surface instability of a laboratory plasma expanding in a magnetic field. The plasma cloud was photographed at a rate ~1 frame/μsec as it dispersed. The plasma temperature was 10 eV, the cloud's total mass was in the range 10^{-7}-10^{-6} g (10^{15}-10^{16} Cu atoms), and its characteristic size was 3 cm. For a collision cross section 10^{-16} cm^2 (the Coulomb cross section is close to the gas-kinetic cross section at this temperature), the particle mean free path is 10^2 cm. At a magnetic induction B = 0.77 T, the Larmor radius reached ~0.5 cm. It is seen that the experiment can be described within the framework of the model of a collisionless plasma with allowance for gyroviscosity that has been adopted here.

Let us estimate the Reynolds number for these parameters:

$$\mathrm{Re} = R_0 \sqrt{\frac{R_0 (p_\perp + p_\parallel)}{\rho R_b} \frac{2\rho\omega_i}{p_\perp}} \approx 2R_0 \sqrt{\frac{2\rho}{p}}\, \omega_i \approx 2\sqrt{2}\, \frac{R_0}{R_i} \approx 17.$$

In accordance with (4.4), the wave number corresponding to the highest growth rate is ~6-7. About that many flutings are indeed observed in the photographs in [4] (Fig. 6). Analyzing the sequence of cloud photographs (corresponding to 9, 12, and 13 μsec after the onset of dispersal), one can note the migration of flutings over its surface across the magnetic field. The fluting migration obeys the rule established above.

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